

### 3.14. Formal Semantics: Conjunctions

Conjunctions are the second type of molecular sentence in the formal language, the product of the third construction rule.

3. If  $\bullet$  and  $\blacktriangle$  are formal sentences, then  $(\bullet \wedge \blacktriangle)$  is a formal sentence.

As always, the semantic rule will shadow the construction rule step for step.

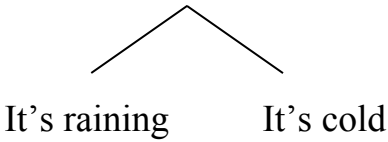
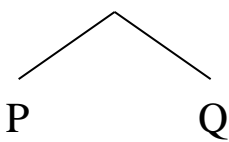
By way of concrete example, we extend the previous English example to include two subject matter sentences, with translation table, and form an English conjunction.

**P:** It's raining

**Q:** It's cold

It's raining and it's cold     $(P \wedge Q)$

The formal conjunction is constructed like so.

<p>It's raining <b>and</b> it's cold.</p>  <p>It's raining      It's cold</p>	<p><math>(P \wedge Q)</math></p>  <p>P                  Q</p>
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The truth table for the conjunction repeats this construction, horizontally.

P	Q	$(P \wedge Q)$
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With two sentence letters, this truth table calls for four valuations.

$$4 = 2 \times 2$$

<b>P</b>	<b>Q</b>
1	1
1	0
0	1
0	0

Since the wedge is meant to model “and” in the formal language, an English conjunction serves as our guide here.

The first valuation marks a case where the sentences “It’s raining” and “It’s cold” are both true. In such a situation it is **true** to say “It’s raining and it’s cold”.

<b>P</b>	<b>Q</b>	<b>(P ∧ Q)</b>
1	1	<b>1</b>
1	0	
0	1	
0	0	

The second valuation presents a situation where it’s true to say “It’s raining” but where it’s false that “It’s cold” – say, a hot rainy summer day. In such a situation the sentence “It’s raining and it’s cold” is **false**.

<b>P</b>	<b>Q</b>	<b>(P ∧ Q)</b>
1	1	1
1	0	<b>0</b>
0	1	
0	0	

With the third valuation we face a situation where it's false that it's raining, but true that it's cold – for instance, a clear winter day. Here the sentence “It's raining and it's cold” is again false.

P	Q	$(P \wedge Q)$
1	1	1
1	0	0
0	1	<b>0</b>
0	0	

The fourth valuation offers a situation where it's false that it's raining, and false that it's cold – a warm, cloudless summer day, for example. It is certainly false here to claim “It's raining and it's cold”.

P	Q	$(P \wedge Q)$
1	1	1
1	0	0
0	1	0
0	0	<b>0</b>

And while we used a specific sentence to think this through, the same pattern will hold for any conjunction, whatever its subject matter. The general semantic rule for conjunctions reads like so.

### Conjunction Rule

●	▲	$(\bullet \wedge \blacktriangle)$
1	1	1
1	0	0
0	1	0
0	0	0

Conjunctions are rather hard to please: **a conjunction is only true when both its parts are true.**